Optimal Subsidy Policy and Missing Products under Monopolistic Competition

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Abstract

Granting a subsidy on a socially preferable product serves to promote that product not only by lowering its consumer price, but also by inducing withdrawal of the other products from the market. That is, the subsidy causes a problem of missing products. The optimal subsidiary policy proposed by this paper shows that the subsidy amount increases with a positive externality of the preferable product. Although this subsidiary policy promotes the socially preferable product, the reduced variety of products mitigates competitive pressure on remaining products, meaning that this policy ultimately raises the price of the socially preferable product. Because this increases in price requires a greater subsidy, the social cost of funding the subsidy increases. Therefore, granting an excessive subsidy on the product may bring about a higher product price by lowering competitive pressure due to missing products, and may ultimately worsen social welfare.

Key Words : Subsidy, monopolistic competition, horizontal product differentiation, promotion policy, missing products.

1. Introduction

The purpose of this paper is to analyze a subsidiary policy aiming to promote a socially preferable product, and it discusses the problem of decreasing product variety caused by the subsidy—that is, the problem of missing products. Granting a subsidy on a product enables society to promote the preferable product not only by its lowering consumer price, but also by inducing withdrawal of other products from the market. However, such withdrawal, that is, missing products, reduces the pressure of market competition on the remaining products and increases these products' prices. The problem of missing products also serves to deter potential new products from entering the market; this hinders product innovation. Thus, when we discuss subsidy policies, it is important to consider the problem of missing products. However, to the author's knowledge, no paper has yet addressed the problem of missing products and subsidy policies.

There are similar problems to that of missing

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products, for example, the problem of the missing middle and the problem of notches. First, the problem of missing middle occurs when favorable tax treatments are endowed on small-size firms under a taxation policy in which the tax amounts are determined by firm size. Such a taxation approach reduces small-size firms' incentive to expand, and as a result, the number of middle-sized firms remains relatively small. Because such taxation structure inhibits small-sized firm from expansion, effective economic growth cannot be attained (Chetty et al., 2011; Dharmapala et al, 2011). That is, if preferential tax treatment is bestowed on a certain target, then, because growth of such target may be restrained, such special treatment may not be able to improve social welfare.

Second, the problem of notches occurs when product characteristics cross a tax bracket threshold and the amount of tax changed discontinuously (see Slemrod, 2013). Because notches exert strong effects on product development, governments can utilize taxation with notches strategically to improve social welfare. In this regard, Arakawa (2014) analyzes the effects of commodity taxation with tax brackets under a vertically differentiated multiproduct monopoly. He shows that a government can improve the quality of all products by inducing the lowest quality product to be bounded on threshold of a tax bracket and moving the tax threshold in the direction of improving product quality. In the automobile industry, for example, Sallee and Slemrod (2012) analyzed fuel economy policies in the U.S. and Canada and showed that many automobiles are produced on the socially preferable sides of tax brackets. Ito and Sallee (2014) analyze the relationship between tax brackets and distribution of automobiles weights in the Japanese automobile industry. They show that tax brackets increase automobile weights, resulting in worse fuel economy and increased damage in automobile accidents. Thus, the development of new cars is strongly affected by notches under taxation with tax brackets, and then the new car is developed on the preferable side of the tax bracket. However, from another perspective, such taxation serves to deter development on the other side of the tax bracket. Furthermore, though it depends on the taxation system settings, socially preferable innovation may be inhibited.[1)] Therefore, the problem of notches shares aspects with the problem of missing products.

In this paper, we use a circular city type of monopolistic competition model based on that of Salop (1979) to (1) analyze effects of a subsidiary policy on social welfare and withdrawal level of products from market, and (2) discuss the problem of missing products under a subsidy policy. An optimal subsidy policy proposed in the present paper counsels granting higher subsidies on more socially preferable products. This policy induces withdrawal of other products, and hence product variety in the market decreases. Although such a measure promotes the socially preferable product more effectively, because decreased product variety lowers market pressure from competition among products, the price of the socially preferable product increases. Thus, higher subsidies are required, and as a result, the social cost of funding the subsidy increases. Therefore, an excessive subsidy decreases product variety more than necessary and raises prices of remaining products due to reduced market pressure arising from product variety. The problem of missing products within the subsidy policy may worsen social welfare.

This paper is organized as follows. In the next section, we show the model. In section 3, we derive the equilibrium for a given subsidy amount. In Section 4, we define the social welfare function and obtain the first-best allocation. In Section 5, we obtain social-welfare-maximizing subsidy levels under the assumption that firms determine product prices. In Section 6, we discuss the problem of missing products with an optimal subsidy policy. Finally, in Section 7, we conclude.

2. Model

The model's settings in this paper are as follows. There are four types of products in the markets, and each product is produced by a different firm. Let the price of Product i produced by firm i be p_i. Products are differentiated in terms of product characteristics, that is, they are differentiated horizontally.[2)] One of the products has socially preferable product characteristics, and government aims to effectively promote the product. However, because consumers do not consider such social desirability, demand for the product is below the socially optimal level. Therefore, the government takes step to grant a subsidy to consumers who buy the product.

We construct a model to analyze a subsidy policy with the above settings using the circular-city-type of monopolistic competition model proposed by Salop (1979). Consumers are uniformly distributed on a circle, which has a perimeter of one. Consumers travel along the circle by incurring quadratic transport costs. That is, consumers traveling a distance x incur the cost tx^2, where t is a transportation parameter. A consumer buys one unit of the product from which he obtains the largest net surplus; he does not buy any products if no product gives him a positive net surplus.

Firms are located around the circle and incur only marginal cost c for producing the product. Because we assume here a short-run situation, we do not consider fixed costs. Within a competitive environment, only firms that obtain positive profits remains in the market, while others withdraw. While all firms compete with each other, each firm directly competes with both sides of neighborhoods.[3)]

Our model is organized as the following three-stage game: in the status quo, there are four products in the market. In the first stage, the government decides the amount of subsidy s. In the second stage, each firm decides whether to remain in the market or not. In the third stage, remaining firms decide product prices. Here we assume that product characteristics, that is, the locations of firms, are determined exogenously. More precisely, the four products in the status quo are located symmetrically, i.e., equidistantly around the circle. The locations of the remaining products in the second stage are assumed to be unchanged. That is, because we consider a short-run situation, granting a subsidy does not affect product development.

The government grants subsidy s to a consumer who buys one unit of product. That is, for each purchased product, a predetermined amount of subsidy is granted to purchasers. In this sense, the subsidy can be considered as subsidy version of a specific commodity tax.[4)] Let us assume that Product 1 has a socially preferable characteristic, and the government aims to promote the product by granting a subsidy.

3. Equilibrium

In this section, we obtain an equilibrium given the amount of subsidy and number of products. That is, we analyze the third stage of the game.

3.1 Case of four products

Demand levels for each product are determined by locations of consumers who are indifferent between adjacent products. Let x_{ij} be a location of a consumer who is indifferent between Products i and j, and it is measured from Product i. Solving following equations simultaneously

$$
\begin{aligned}
&p_1 - s + tx_{12}^2 = p_2 + t(1/4 - x_{12})^2,\\
&p_2 + tx_{23}^2 = p_3 + t(1/4 - x_{23})^2,\\
&p_4 + tx_{34}^2 = p_3 + t(1/4 - x_{34})^2,\\
&p_1 - s + tx_{14}^2 = p_4 + t(1/4 - x_{14})^2,
\end{aligned}
$$

we obtain locations of indifferent consumers as follows:

$$x_{12}=\frac{-16p_1+16p_2+16s+t}{8t},$$

$$x_{23}=\frac{-16p_2+16p_3+t}{8t},$$

$$x_{34}=\frac{16p_3-16p_4+t}{8t},$$

$$x_{14}=\frac{-16p_1+16p_4+16s+t}{8t}.$$

Based on the above locations, we obtain demand for each product as follows:

$$D_1=x_{12}+x_{14},\ D_2=\frac{1}{4}-x_{12}+x_{23},$$

$$D_3=\frac{1}{2}-x_{23}-x_{34},\ D4=\frac{1}{4}-x_{14}+x_{34},$$

where D_i is demand for Product i. Firm i's profit π_i is obtained as

$$\pi_i=(p_i-c)D_i. \tag{1}$$

By differentiating profit functions with respect to product prices for each firm, equating those equations to zero, and solving them simultaneously, we obtain following solutions:

$$p_1=c+\frac{5s}{12}+\frac{t}{16},\ p_2=p_4=c+\frac{t}{16}-\frac{s}{6}, \tag{2}$$

$$p_3=c+\frac{t}{16}-\frac{s}{12},$$

$$x_{12}=x_{14}=\frac{1}{8}+\frac{5s}{6t},\ x_{23}=x_{34}=\frac{1}{8}+\frac{s}{6t}.$$

From the above solutions, we find that as the subsidy on Product i increases, the price of Product 1 increases, while prices of other products decrease. Furthermore, when the subsidy amount increases, demand for Products 1 and 3 increases, while that for other products decreases. In sum, as the subsidy amount rises, Firms 2 and 4 face difficulties in setting a product price above the marginal cost; they also struggle to obtain demand for their products. In the following, we derive a condition for existence of the four products.

First, let us consider the conditions needed for Products 2 and 4 to have prices higher than marginal cost. From equation (2), we find that the condition is as follows:

$$s<\frac{3t}{8}. \tag{3}$$

Next, let us consider the conditions for Products 2 and 4 to obtain market demand. In terms of Product 1, because $D_1=1/4$ if $s=0$, and $\partial D_1/\partial s=5/(3t)>0$, we do not have to consider the conditions. In terms of Product 2, $D_2=1/4$ if $s=0$, and $\partial D_2/\partial s=-2/(3t)<0$. Furthermore, $D_2=0$ if $s=3t/8$. Thus, the condition for Product 2 to obtain market demand is same as in equation (3). In terms of Product 3, $D_3=1/4$ if $s=0$, and $\partial D_3/\partial s=-1/(3t)<0$. Furthermore, $D_3=0$ if $s=3t/4$. Therefore, the condition for Product 3 to obtain market demand is $s<3t/4$. Finally, in terms of Product 4, the condition of obtaining market demand is same as that of Product 2.

To summarize the above, the condition of existence for the four products is given by equation (3). If this condition cannot be met, Products 2 and 4 have to withdraw from the market, and only Products 1 and 3 remain. In the status quo, that is, when no subsidy is granted, this condition is satisfied and the market accommodates four products. In other words, when the subsidy amount is relatively small, then because competitive advantage of Product 1 is not large, all products garner market demand. However, when the subsidy amount is relatively large, two products withdraw from the market under equation (2), and only two products remain in the market.

3.2 Case of two products

Let us consider the case when the subsidy amount does not meet condition (3), that is, when it is relatively large, the number of products may be two. By using similar considerations to those employed in the four-product case, we obtain solutions with firm profit-maximizing behaviors as follows:

$$p_1 = c + \frac{t}{4} + \frac{s}{3},\ p_3 = c + \frac{t}{4} - \frac{s}{3}, \tag{4}$$
$$x_{13} = \frac{1}{4} + \frac{s}{3t}.$$

These solutions show that as the amount of subsidy increases, the price of Product 1 increases whereas that of Product 3 decreases. Furthermore, by increasing the subsidy amount, demand for Product 1 increases while that of Product 3 decreases. Thus, as with the four-product case, if the amount of subsidy increases, it is hard for the firm producing Product 3 to set a price higher than the marginal cost as well as to obtain market demand.

Let us obtain the condition where Product 3 can remain in the market. First, the condition where Product 3 can set a price higher than marginal cost is

$$s < \frac{3t}{4}. \tag{5}$$

Next, the condition where Product 3 obtains market demand is derived as follows. In terms of Product 3, $D_3 = 1/4$ if $s = 3t/8$, and $\partial D_3 / \partial s = -2/(3t) < 0$. Furthermore, $D_3 = 0$ if $s = 3t/4$. Thus, the condition is same as in equation (5). In sum, if condition (5) is satisfied, the market is able to accommodate two products.

However, even if equation (5) is satisfied, there are possibility that Products 2 and 4 remain in the market. In this case, because four products exist in the market, prices of equation (4), derived under an assumption of two products, are not valid. Thus, we have to obtain condition such that only two products can remain. To do this, we have to check whether Products 2 and 4 remain in the market under equation (4).

Assuming that the market accommodates four products, and strategies of Product 1 and 3 are given in equation (4), we obtain the solution by maximizing profits of Products 2 and 4 as follows:

$$p_2 = p_4 = c + \frac{5t}{32} - \frac{s}{4},$$
$$x_{12} = x_{14} = -\frac{1}{16} + \frac{5s}{6t},\ x_{23} = x_{34} = \frac{5}{16} - \frac{s}{6t}.$$

These solutions show that when the amount of subsidy increases, prices and market demand of Products 2 and 4 decreases. Thus, we find that as the subsidy amount increases, it is difficult for either Product 2 or 4 to remain in the market.

Let us obtain the condition where the market accommodates two products. First, the condition where Products 2 and 4 are not be able to set their prices higher than marginal cost, that is, the condition of withdrawal for Products 2 and 4 is

$$s \geq \frac{5t}{8}. \tag{6}$$

Next, let us obtain a condition where only two products receive market demand. In terms of Product 1, because $D_1 = 1/2$ if $s = 3t/8$, and $\partial D_1 / \partial s = 5/(3t) > 0$, we do not have to consider this case. In terms of Product 2, $D_2 = 1/4$ if $s = 3t/8$, and $\partial D_2 / \partial s = -1/t < 0$. Furthermore, $D_2 = 0$ if $s = 5t/8$. Thus, the condition where Product 2 obtain market demand is $s < 5t/8$. In terms of Product 3, because $D_3 = 0$ if $s = 3t/8$, and $\partial D_3 / \partial s = 1/(3t) > 0$, we do not have to consider this case. In terms of Product 4, the condition is same as that of Product 2. Therefore, we find that the condition where only two products remain in the market is $s \geq 5t/8$. In this case, unique equilibrium is given in equation (4). Thus, when $s \geq 5t/8$, there are two products with equation (4).

However, when $s < 5t/8$, because subsidy amount is small, causing the competitive advantage of Product 1 to decline, the market accommodates four products. Let us consider a case when $s \in [3t/8,\ 5t/8)$. As already discussed above, under equation (4) obtained with the assumption of two products, because Product 2 and 4 remain in the market, it is difficult for Product 3 to obtain large market demand. Especially when $s = 3t/8$, the market demand for Product 3 is zero. In this case, if Product 3 set a price lower than that indicated by equation (4), and holding market demand for Products 2 and 4 to zero, Products 2 and 4 can be deterred from remaining in the market and Product 3 can obtain market demand.

Reaction functions of Products 2 and 4 given the prices of Products 1 and 3 are

$$p_2 = p_4 = \frac{1}{32}(16c + 8p_1 + 8p_3 - 8s + t).$$

The market demand of Products 2 and 4 are obtained as follows:

$$D_2 = D_4 = \frac{-16c + 8p_1 + 8p_3 - 8s + t}{8t}.$$

To keep these demand levels equal to zero, prices of Products 1 and 3 have to satisfy the following condition:

$$-16c + 8p_1 + 8p_3 - 8s + t = 0. \tag{7}$$

If not satisfied, Products 2 and 4 remain in the market and the profits of Products 1 and 3 decrease. Under this condition, both Products 1 and 3 do not have incentive to change their prices. Thus, combinations of prices satisfying this condition can be considered equilibrium prices. Although there are innumerable combinations of equilibrium prices, let us obtain a range of prices for Product 3.

First, the highest price is obtained as follows. For the two-product case, a consumer who is indifferent between the two products is

$$x_{13} = \frac{-4p_1 + 4p_3 + 4s + t}{4t},$$

and market demand for the two products is

$$D_1 = 2x_{13},\ D_3 = 1 - 2x_{13}.$$

By solving $D_3 = 0$, the highest price of Product 3 is obtained as

$$p_3 = p_1 - s + \frac{t}{4}.$$

Furthermore, by solving equation (7) and the above prices simultaneously, we have

$$p_1 = c + s - \frac{3t}{16},\ p_3 = c + \frac{t}{16}. \tag{8}$$

Second, the lowest price of Product 3 is same as marginal cost. In this case, from equation (7), we have

$$p_1 = c + s - \frac{t}{8},\ p_3 = c.$$

Because the market demand levels in this case are

$$D_1 = \frac{3}{4},\ D_2 = \frac{1}{4},$$

we find that the market demand cannot be affected by the subsidy amount. In sum, a range of prices for Product 3 are obtained as follows:

$$p_3 \in [c,\ c + t/16]. \tag{9}$$

Though combinations of equilibrium prices are restricted by the price range of equation (9), innumerable combinations of equilibrium prices remain. Because it is difficult to conduct advance research in this situation, we assume that the possible combinations of equilibrium prices are as in equation (8), that is, we assume that Product 3 can set the highest price. The reason for this is as follows: we consider the situation in which the subsidy granted by the government changes the market structure from one with four products (i.e., the status quo) to a market with two products. While the prices in the status quo are $p_1 = p_2 = c + t/16$, it is rational to assume that the prices do not change due to the subsidy. In this case, the changed price is the highest price according to equation (9). In other words, even if the market structure changes due to the subsidy, firms' strategies are maintained as much as possible.

In sum, there exists a unique equilibrium with two products; when $s \in [3t/8,\ 5t/8)$, the prices are determined by equation (8), and when $s \geq 5t/8$, the prices are determined by equation (4).

3.3 Case of one product

When the subsidy amount satisfies equation (6), that is, when it is relatively large, the market may accommodate only Product 1. In other words, when the subsidy amount is large, then, because the competitive advantage of Product 1 is too large, it is difficult for Product 3 to set a price higher than marginal cost and to obtain market demand. However, if Product 1 attempts to remain in the market alone and to set a significantly higher price, Product 3 can remain in the market. Thus, we maximize Product 1's profit under the condition where Product 3's profit is zero. However, even if Product 3 is expelled from the market, Products 2 and 4 may remain in the market. Therefore, we also have to consider the condition where Products 2 and 4 do not remain in the market.

First, let us consider the situation when Product 3 leaves the market. The reaction function of Product 3 given the price of Product 1 is

$$p_3 = \frac{1}{8}(4c + 4p_1 - 4s + t).$$

The price of Product 1 that maintains demand for Product 3 as zero is

$$p_1 = c + s - \frac{t}{4}. \tag{10}$$

This is the highest price of Product 1 to Product 3 out of the market.

Next, let us consider the case when Products 2 and 4 leave the market. The reaction functions of Products 2 and 4 given the price of Product 1 are

$$p_2 = p_4 = \frac{1}{112}(80c + 32p_1 - 32s + 5t).$$

The price needed for Product 1 to keep Products 2 and 4 out the market is

$$p_1 = c + s - \frac{5t}{32}. \tag{11}$$

This is the highest price that can be charged for Product 1 to keep Products 2 and 4 out the market. Because equation (10) is less than equation (11), the price of Product 1 where the market accommodates only Product 1 is given by equation (10).

The following proposition is a summary of the above.

Proposition 1. Equilibrium in the third stage is as follows: (i) When $s < 3t/8$, the market accommodates four products and product prices are given by equation (2); (ii) when $s \in [3t/8, 5t/8)$, the market accommodates two products and prices are given by equation (8); (iii) when $s \in [5t/8, 3t/4)$, there are two products in the market, and their prices are determined by equation (4); and (iv) when $s \geq 3t/4$, only one product remain in the market and its price is given by equation (10).

This proposition shows that if the subsidy amount and competitive advantage of Product 1 increase, the other products are induced to quit the market. It is not difficult to understand this result, and we find that the subsidy serves to promote the socially beneficial product. However, note that when the subsidy amount increases and the other products withdraw the market, price of Product 1 increases discontinuously.[5] This is because, depending on price increase, granting the subsidy may worsen social welfare. This point, i.e., the problem of missing products, will be discussed in Section 6.

4. Social welfare

We define social welfare (SW) as the sum of surpluses from both consumers and firms. Because social welfare differs depending on the number of products, we analyze social welfare for each number of products. Furthermore, we obtain social welfare under the status quo and also derive first-best allocation attained by a social planner.

First, let us consider the scenario when the market accommodates four products. In this case, consumer surplus (CS) is obtained as follows:

$$CS = \int_0^{x_{12}} (V - p_1 + s - tx^2)\,dx +$$
$$+ \int_0^{x_{14}} (V - p_1 + s - tx^2)\,dx +$$
$$+ \int_0^{1/4 - x_{12}} (V - p_2 - tx^2)\,dx +$$
$$+ \int_0^{x_{23}} (V - p_2 - tx^2)\,dx + \int_0^{1/4 - x_{23}} (V - p_3 - tx^2)\,dx +$$
$$+ \int_0^{1/4 - x_{34}} (V - p_3 - tx^2)\,dx +$$
$$+ \int_0^{1/4 - x_{14}} (V - p_4 - tx)\,dx +$$
$$+ \int_0^{x_{34}} (V - p_4 - tx)\,dx,$$

where V is the utility that a consumer derives from one unit of product, and a is a parameter representing a degree of positive externality exerted by Product 1. The amount of positive externality (PE) is defined as

$$PE = a(x_{12} + x_{14}).$$

Firms', surpluses coincide with sum of firms' profits, that is, the sum of equation (1). Let us assume that fund of the subsidy is a social cost (SS)[6]:

$$SS = s(x_{12} + x_{14}).$$

In sum, social welfare is defined as follows:

$$SW = CS + \sum\nolimits_i \pi_i - SC + PE. \tag{12}$$

We omit the derivation of SW for the two and one product cases because they are derived similarly to the four-product case.

To measure the effect of the subsidy, we have to compare the situation with and without the subsidy. Thus, we derive SW without the subsidy in the following. Note that the status quo has four products. Substituting equation (2) into equation (12), and setting the subsidy to zero, we have

$$SW = \frac{a}{4} - c - \frac{t}{192} + V.$$

Let us obtain the first-best allocation derived by a social planner as a benchmark. In the first-best allocation, the social planner maximizes SW by setting all product prices equal to marginal cost. First, let us consider the case of four products. In this case, the social-welfare-maximizing amount of subsidy is obtained as

$$s = a.$$

The demand for products are obtained as

$$D_1 = \frac{1}{4} + \frac{4a}{t},\ D_2 = D_4 = \frac{1}{4} - \frac{2a}{t},\ D_3 = \frac{1}{4}. \tag{13}$$

From equation (13), we find that when $a \geq t/8$, the market demand for Products 2 and 4 becomes zero. That is, if the positive externality exerted by Product 1 is relatively larger than the transport parameter, it is socially preferable to induce Products 2 and 4 to withdraw from the market. Thus, we find that the first-best allocation with four product exists if $a < t/8$. In this case, we have

$$SW = \frac{a}{4} - c + \frac{2a^2}{t} - \frac{t}{192} + V. \tag{14}$$

Next, let us consider the case when the market accommodates two products. The subsidy amount that maximizes SW is

$$s = a$$

The market demand levels in this case are

$$D_1 = \frac{1}{2} + \frac{2a}{t},\ D_3 = \frac{1}{2} - \frac{2a}{t}. \tag{15}$$

From equation (15), if $a \geq t/4$, we find that the market demand for Product 3 is zero. That is, when the positive externality from Product 1 is relatively large, we find that it is socially preferable for only Product 1 to remain in the market. Thus, the first-best allocation with two products exists if $a < t/4$. In this case, we have

$$SW = \frac{a}{2} - c + \frac{a^2}{t} - \frac{t}{48} + V. \tag{16}$$

Finally, when the market accommodates only Product 1, SW is obtained as follows[7]:

$$SW = a - c - \frac{t}{12} + V. \tag{17}$$

While we have considered the number of product as a given, if social planner can control the number of products, we have to derive the socially preferable number of products. By analyzing the relationship between number of products and social welfare, we have the following proposition.

Proposition 2. In the first-best allocation, all product prices are set as marginal cost and the subsidy amount is same as the degree of Product 1's positive externality. Furthermore (i) when $a \in [0,\ t/8)$, the market accommodates four products; (ii) when $a \in [t/8,\ t/4)$, there are two products in the market; and (iii) when $a \geq t/4$, only one product can remain in the market.

Proof. First, by comparing the first-best with four products [equation (14)] and that with two products [equation (16)], we find that when $a < t/8$, equation (14) is larger than equation (16). Next, by comparing the first-best with four products [equation (16)] and that with two products [equation (17)], we find that when $a < t/4$, equation (16) is larger than equation (17). Q.E.D.

The first-best allocation shows that as Product 1's positive externality of increases, it is socially preferable to decrease the number of products remaining in the market. That is, in the relationship between degree of positive externality and consumer transport costs, even if the socially preferable product is not suitable for consumer preferences, it is socially preferable to induce consumers to buy the socially preferable product.

5. Optimal subsidy policy

In the first-best allocation, the social planner sets product prices equal to the marginal cost. However, when product prices are set by the firms, because firms attempt to set their prices higher than the marginal cost, even if the government grants a subsidy to control the market, it is difficult to achieve the first-best allocation. Thus, to measure the effect of a subsidy policy, we have to consider a realistic situation, that is, the second-best allocation where the firms determine their prices while the government grants a subsidy to consumers.

5.1 Subsidy policy for a given number of products

The market structure differs according to the number of products existing in that market. Thus, as the first-best allocation, we first analyze the subsidy policy for a given number of products, and next, we obtain the second-best allocation by comparing the social welfare, that is, we obtain the optimal subsidy policy.[8)]

(a) Case of four products

Substituting equation (2) into equation (12), and differentiating the obtained equation with respect to price, we have following subsidy amount:

$$s = \frac{30a}{13}. \tag{18}$$

Prices in this case are as follows:

$$p_1 = c + \frac{t}{16} + \frac{25a}{26},\ p_2 = p_4 = c + \frac{1}{16} - \frac{5a}{13},$$

$$p_3 = c + \frac{t}{16} - \frac{5a}{26}.$$

These results show that as the positive externality increases, prices of all products except Product 1 decrease. The condition where Products 2 and 4 can set their prices higher than marginal costs is

$$a < \frac{13t}{80}. \tag{19}$$

Thus, only when the positive externality is sufficiently lower than the transport parameter can the market accommodates four products. From equations (18) and (19), we find that the condition in which four products can be accommodated in the market presented in Proposition 1, $s<3t/8$, is satisfied. Thus, if equation (19) is satisfied, the subsidy amount of equation (18) can be equilibrium. In this case, we have

$$SW=\frac{a}{4}-c+\frac{25a^2}{13t}-\frac{t}{192}+V. \tag{20}$$

Note that if equation (19) is not satisfied, there are no equilibria with four products under equation (18).

(b) Case of two products

From Proposition 1, the equilibria are divided into two types according to the subsidy amount. First, let us consider when $s\in[3t/8,\ 5t/8)$. Substituting equation (8) into the SW function, we have

$$SW=a-c-\frac{t}{12}+V. \tag{21}$$

Because equation (21) does not include s, we find that the subsidy amount does not affect social welfare.

Next, let us consider the case when $s\in[5t/8,\ 3t/4)$. By substituting prices into the SW function and differentiating obtained equation with respect to the subsidy amount, we have

$$s=3a. \tag{22}$$

Prices in this case are

$$p_1=c+\frac{t}{4}+a,\ p_3=c+\frac{t}{4}-a.$$

The condition in which Product 3 remains in the market is

$$a<\frac{t}{4}. \tag{23}$$

From equations (22) and (23), we find that the condition in which two products remain in the market, i.e., the one presented by Proposition 1, $s\in[5t/8,\ 3t/4)$, is satisfied. Thus, equation (22) can be an equilibrium. In this case, we have

$$SW=\frac{a}{2}-c+\frac{a^2}{t}-\frac{t}{48}+V. \tag{24}$$

Note that if equation (23) is not satisfied, there are no equilibria with two products under equation (22).

(c) Case of one product

From Proposition 1, equilibrium price of Product 1 is given by equation (10). In this case we have

$$SW=a-c-\frac{t}{12}+V.$$

5.2 Optimal subsidy

Here, we obtain optimal subsidy policy for the situation in which the government can control the number of product in the market. From the above analysis, we have following proposition.

Proposition 3. The optimal subsidy policy is as follows: (i) when $a\in[0,\ 13t/80)$, four products remain in the market with $s=30a/13$; (ii) when $a\in[13t/80,\ t/4)$, the market accommodates two products with $s=3a$; and (iii) when $a\geq t/4$, only one product remains in the market with $s\geq 3t/4$.

Proof. First, by comparing equations (20) and (21), equation (20) is larger under the range of equation (19). Next, by comparing equations (21) and (24), equation (24) is larger under the range of equation (23). Q.E.D.

Let us consider the optimal subsidy case in more detail. As the positive externality increases, the subsidy amount increases, and the price of Product 1 also increases. This coincides with our intuition. However, because not all but only some portion of the subsidy contributes to the profit of Firm 1, the subsidy amount must rise. That is, if the government grants a subsidy for purchases of a socially preferable product, because the product's producer raises the price by taking the subsidy for granted, the government will have to grant a larger subsidy. However, the problem further impacts the market as follows.

If the competitive advantage of Product 1 increases, then because other products have lost market power and they are induced to withdraw from the market. In addition, if the positive externality of Product 1 increases, it is socially preferable to promote Product 1 and it may also be preferable for Products 2 and 4 to withdraw from the market, then at a glance, no problem arises from the subsidy giving competitive advantage to Product 1. However, the other products create a competitive environment and exert market pressure to lower prices. Thus, if the other products are placed at a disadvantage and forced to withdraw from the market, the remaining products can increase their prices. From the above reasoning, we find that the subsidy has two functions: (i) promote subsidized products, and (ii) raise the subsidized products' prices.

The optimal subsidy policy presented in Proposition 3 considers the above two functions and shows the condition of decreasing the number of products to effectively promote the socially preferable product. That is, when a positive externality is relatively small, by keeping the number of products as the status quo, the government can improve the social welfare in a competitive environment. On the other hand, when the positive externality is relatively large, the government improves social welfare not by utilizing a competitive environment featuring many products, but by forcing the other products to withdraw from the market so that only the socially preferable product remains. When the positive externality has an intermediate value, then while the government decreases the number of products so as to promote socially preferable products, the competitive environment is still utilized with the two remaining products. That is, in this case, the government improves social welfare by balancing these two functions.

6. Discussion

When the government executes a subsidy policy in the real world, it may be difficult to balance those two functions. Because the government may concentrate on promoting socially preferable products, it may ignore social views regarding the cost-effectiveness of promoting the products. That is, if the government justifies driving rival products from the market and also justifies granting a subsidy to purchasers of preferable product, it may ignore the problem of a large price increase in the subsidized product. In this case, the government may have to grant an excessive subsidy. Furthermore, this causes other products to withdraw from the market, and the remaining products obtain excessive profit. As a result, social welfare may not be improved. In some cases, social welfare may in fact deteriorate compared to that of the status quo. This can be easily understood by the following proposition.

Proposition 4. If the government grants a subsidy when the positive externality is less than the range presented in Proposition 3 while the amount of subsidy is that given in Proposition 3, social welfare may decline compared to that of the status quo.

Proof. For example, when the government grants a subsidy $s=3a$ to create a market with two products, if $a\in[0,\ (-1+\sqrt{2})t/8(=0.0518t))$, which is less than the range $a\in[13t/80(=0.1625t),\ t/4)$, social welfare is lower than that of the status quo. Q.E.D.

As already have explained, if the government grants an excessive subsidy, the number of products in the market may significantly decreased, while prices of the remaining products increase excessively. Furthermore, if an excessive subsidy is granted, entry of potential new products into the market may be deterred; this causes a substantial spread of the socially preferable product. It is possible to analyze the withdrawal of products caused by the subsidy empirically using market data. However, it is difficult to analyze the deterrence facing potential new products into the market because no data can be collected in terms of such a situation. From the above, it is necessary to pay full attention to a subsidy policy aiming to improve social welfare by promoting preferable products because the government may focus on the spread of socially preferable products without considering the welfare effect of such subsidies. That is, the government has to keep in mind the problem of missing products.

Arakawa (2011) discusses welfare effects of the tax break system for Kei Cars in Japan. He shows that socially preferable products tend to be undersupplied. He proposes a solution for this problem whereby levying small amount of tax on socially preferable products enables the government to improve social welfare. Furthermore, he also shows that tax breaks for special category products such as Kei Cars in Japan may erode social welfare. This occurs because a tax break for a certain product category may undermine or eliminate incentives to improve the product (i.e., product innovation), or deter potential new products belonging to the other categories. This situation has to be analyzed from a viewpoint of the problem of missing products.

7. Conclusion

This paper analyzed the optimal subsidy policy under monopolistic competition. The optimal subsidy has to rise in conjunction with the degree of positive externality exerted by the socially preferable product. Granting a subsidy induces other products to withdraw from the market, creating a situation where the market suffers declining product variety. Thus, while the subsidy has a function of spreading socially preferable products, it also has a function of calming a competitive environment by the decreasing number of products in the market. As a result, the subsidy increases prices of the subsidized product. Therefore, the government has to increases the subsidy amount, leading the social cost of funding the subsidy to rise as well. This result shows that an excessive subsidy decreases the number of products to an excessive extent, and this unnecessarily lowered competitive environment increases the subsidized product's price. That is, the subsidy policy induces the problem of missing products.

This paper utilized the concept of horizontal product differentiation wherein products are differentiated via characteristics. The situation in which purchasers of socially preferable product are subsidized was analyzed. In this case, no products except the subsidized product have socially preferable product characteristics—that is, no difference exists between the other products from a social viewpoint. However, in the automobile industry, the government grants a subsidy on cars with excellent environmental performance, while all cars are differentiated with respect to environmental performance and they can be ranked according to environmental performances. That is, in general, all products are differentiated with respect to their qualities and the government (i) levies a tax, the amount of which reflects product qualities or (ii) grants a subsidy according to the products' qualities. By considering these situations together with the concept of vertical product differentiation, we can discuss a subsidy policy aiming to improve overall product quality in an industry. In this case, similar to this paper, we can discuss the subsidy policy from the viewpoint of the problem of missing products in a vertically differentiated, rather than horizontally differentiated, market. This is a future problem.

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Notes

1) Kleven and Waseem (2013) theoretically analyze relationship between a taxation system with tax brackets and product innovation.

2) Products are horizontally differentiated if they consist of different characteristics, and consumer preferences are reflected in demand for these products. Thus, if different products are sold at the same price, each product has non-zero demand. On the other hand, products are vertically differentiated if they consist of different product qualities, and all consumers have same preference ranking for the products. Therefore, if all products are sold at same price, only the product with highest quality incurs demand.

3) In the monopolistic competition model proposed by Dixit and Stiglitz (1977), all firms compete directly with other firms. Because they do not consider product location, their model is called a non-address model, while the model of Salop (1979) is called an address model.

4) Note that if we assume a subsidy amount determined as a proportion of product price, the conclusion does not change in this setting. For further details, see Salanie (2011) and Arakawa

(2012).

5) We can prove this by substituting the subsidy amount into the equilibrium prices of equations (2), (8), (4), and (10), and comparing the prices.

6) Because the subsidy granted on Product 1 has social value, there may be no problem if the funds used to provide the subsidy is collected from the entire society. Thus, we do not consider this specific method in this paper.

7) Because there is only Product 1 in the market, all consumers buy Product 1 and obtain the subsidy. Thus, the subsidy does not affect consumer behavior and the cost of funds used to provide the subsidy and the social cost cancel each other. Furthermore, because all consumers buy Product 1, the prices of Products 1 and 3 cancel each other. Therefore, the subsidy amount and price of Product 1 do not appear in the SW function.

8) In this paper, we call the optimal subsidy policy a social-welfare-maximizing policy within the framework of a competitive economy in which firms determine product prices.